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A CURVE-FITTING TECHNIQUE FOR THE DETERMINATION OF GAIN AND SATURATION INTENSITY IN HOMOGENEOUSLY-BROADENED GAS LASERS: APPLICATION TO A WAVEGUIDE CO_2 LASER

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A CURVE-FITTING TECHNIQUE FOR THE DETERMINATION OF GAIN AND SATURATION INTENSITY IN HOMOGENEOUSLY-BROADENED GAS LASERS: APPLICATION TO A WAVEGUIDE CO_2 LASER

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ABSTRACT

A least squares curve-fitting algorithm is derived which allows the simultaneous estimation of the small signal gain and the saturation intensity from an arbitrary number of data points relating power output to the incidence angle of an internal coupling plate. The method is used to study the dependence of the two parameters on tube pressure and discharge current in a waveguide CO_2 laser having a 2 mm diameter capillary. It is found that, at pressures greater than 28 torr, rising CO_2 temperature degrades the small signal gain at current levels as low as three milliamperes. Peak small signal gain in the pressure and current range studied is 12.6 db/meter at 20° C. The large values of saturation intensity observed (typically in the range 0.5 to 4.4 kilowatts/cm^2) cannot be explained totally by pressure and radial diffusion effects.

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A CURVE-FITTING TECHNIQUE FOR THE DETERMINATION OF GAIN AND SATURATION INTENSITY IN HOMOGENEOUSLY-BROADENED GAS LASERS: APPLICATION TO A WAVEGUIDE CO_2 LASER

I. INTRODUCTION

Marcatili and Schmeltzer[1] proposed the use of a hollow dielectric waveguide to confine and guide the radiation in a gas discharge thereby taking advantage of the inverse relationship between gain and bore diameter in gaseous lasers.[2] Smith[3] first applie· the technique to helium-neon lasers. Bridges, Burkhardt and Smith[4] and Jensen and Tobin[5] have since reported operation of two independently built waveguide CO_2 lasers. Recently Smith[6] et al. announced that a high pressure waveguide CO_2 laser had been successfully mode-locked. The present authors wish to report the results of measurements made of the unsaturated gain coefficient and saturation power of a water-cooled, flow-type waveguide CO_2 laser at various total gas pressures and discharge currents using a method which makes full use of the gain saturation equations of Rigrod.[7]

II. EXPERIMENTAL DETAILS

The waveguide laser tube used in this experiment is shown in Figure 1. The capillary had an inner bore diameter of 2-mm and an overall length of 12.5 cm. The total tube length was 22 cm with opposing salt Brewster windows at the ends. Excitation was by DC discharge using a hollow cylinder nickel cathode

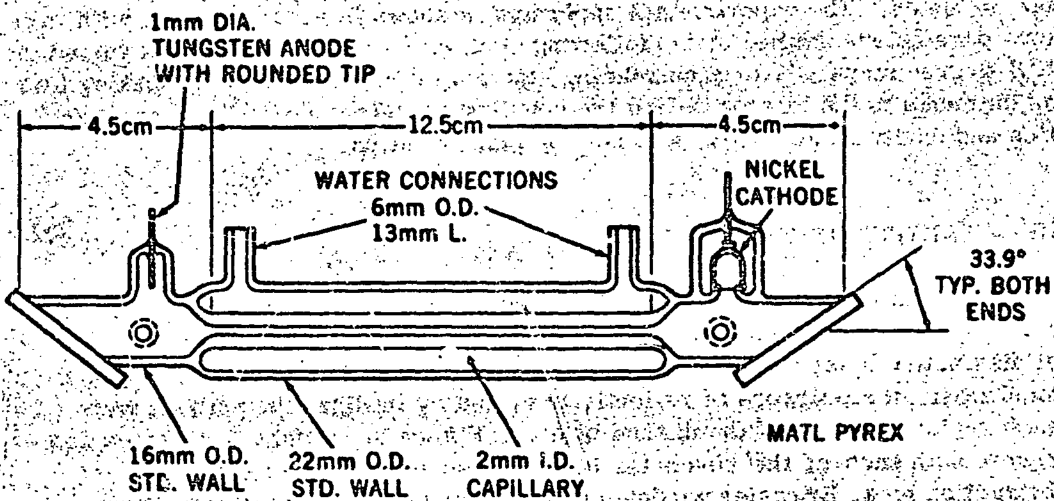

Figure 1. Waveguide laser tube design.

and a 1-mm diameter tungsten anode through a 600 KΩ ballast resistance. The non-optimized gas was premixed at a CO_2 : N_2 : He ratio of 1 : 1 : 4. The gas pressures were observed at three points in the system:

(1) at a control box separated from the tube input by approximately 1.7 meters of 4.25-mm ID tubing;

(2) just prior to the input and

(3) immediately following the tube output.

It was found that the tube pressure varied significantly at each of these points. The pressure differential across the tube was between 7 and 12 torr and increased with increasing average tube pressure. Furthermore, the pressure reading at the input to the tube was reduced by between 3 and 5 torr when the discharge current was turned off while the pressure at the output remained relatively constant. The flow rate was also affected by discharge current. The results of the pressure and flow rate measurements are shown in Table I.

The resonator consisted of two gold-coated mirrors having 52.4 cm radii of curvature and mounted on tri-directional translation stages. The mirrors had angular control as well. The centers of curvature were situated approximately 1.0 cm into the guide from the flared ends since the latter arrangement seemed to give optimum coupling between the resonator and the waveguide. External coupling was achieved by means of a germanium coupling plate 5-mm thick and mounted on an angularly calibrated turntable. The power output from one side of the plate was observed as a function of plate angle using a CRL Model 201 power meter. The experimental apparatus is shown in Figure 2 along with various parameters defined in the following section. The plate angle was varied in half-degree steps. With each reading, the power was optimized by making fine adjustments to the mirror behind the coupling plate to correct for beam translation and cavity length changes caused by plate rotation.

III. ANALYSIS OF THE DATA

The data relating power output to incident angle was interpreted with the aid of a set of theoretical equations derived by Rigrod.[7] Because of a variety of dissipative losses and the inclusion of the germanium plate is the coupling mechanism, the mirrors of reflectivity r_1 and r_2 in Rigrod's analysis were each replaced by one of the dashed boxes in Figure 2. Dissipative losses common to both ends of the resonator include absorption in the mirror surface, diffraction loss, Brewster window scattering and absorption, astigmatism

Table I
Pressure (Torr)

	Discharge ON	Discharge OFF
Control Box	28.1	25.9
Tube Input	24.3	21.5
Tube Output	17.0	16.7
Average Tube Pressure	20.6	19.1
Flow Rate (m^3/sec)	7.62×10^{-5}	7.70×10^{-5}
Control Box	38.1	34.7
Tube Input	33.0	29.3
Tube Output	22.9	22.7
Average Tube Pressure	28.0	26.2
Flow Rate (m^3/sec)	8.33×10^{-5}	8.88×10^{-5}
Control Box	48.1	43.6
Tube Input	41.8	36.7
Tube Output	28.7	28.4
Average Tube Pressure	35.2	32.6
Flow Rate (m^3/sec)	9.72×10^{-5}	9.38×10^{-5}

Capillary volume = $1.57 \times 10^{-6} m^3$.

caused by the Brewster window,[8] and possible reflective losses at the end of the capillary due to the abrupt change in the boundary condition or imperfect coupling between the resonator and the guide. The insertion of the germanium coupling plate introduces additional loss at one end of the cavity. At this end the "effective reflectivity" can be written as

$$r_1 = t_j \, t_b^2 \, r_m \, t_p^2 \, t_r \tag{1}$$

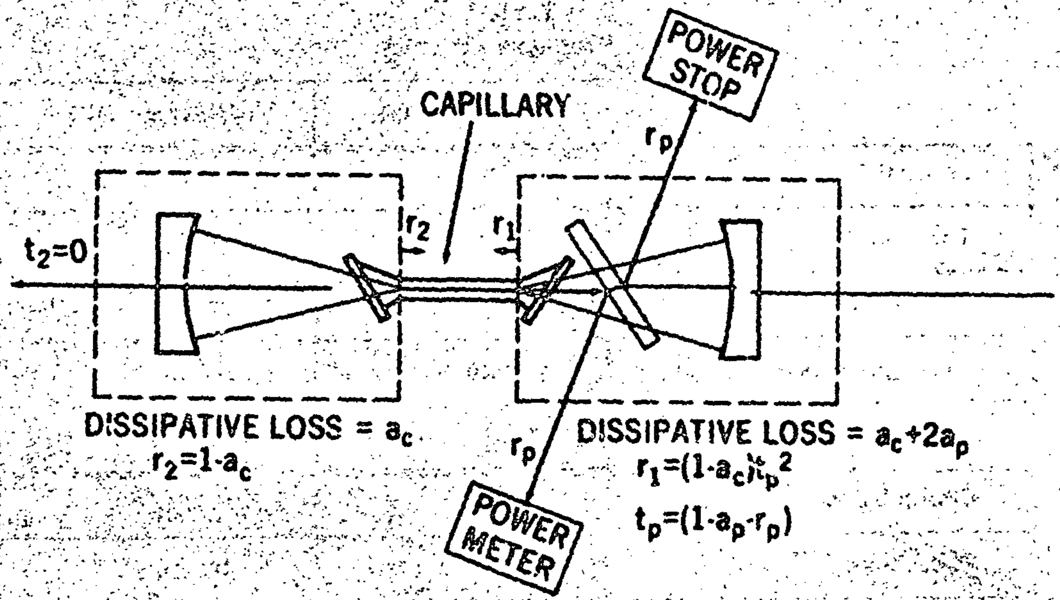

Figure 2. Experimental apparatus and method of analysis: the mirrors of reflectivity r_1 and r_2 in Rigrod's analysis are replaced by the dashed boxes in order to take into account the various dissipative losses. The gain length is assumed equal to the capillary length.

where t_j is the fractional transmission of the power exiting through the capillary-resonator "junction", t_b the transmission through the Brewster window, t_p the transmission of the germanium plate, r_m the reflectivity of the mirror, and t_r the effective coupling of the returning radiation into the guide. It is assumed in the present analysis that the dissipative losses common to both ends of the guide are equal. Hence,

$$r_2 = t_j\, t_b^2\, r_m t_r = (1-a_j)\,(1-a_b)^2\,(1-a_m)\,(1-a_r) \tag{2}$$

where the "a's" are the appropriate fractional losses. If the latter losses are assumed individually small so that second order terms can be ignored, we can write

$$r_2 = 1-a_c \tag{3}$$

where

$$a_c = a_j + 2a_b + a_m + a_r \tag{4}$$

is a net dissipative loss common to both ends of the resonator. We can then rewrite equation (1) as

$$r_1 = (1-a_c)\, t_p^2 = (1-a_c)\,(1-a_p-r_p)^2 \tag{5}$$

where a_p is the additional absorption in the germanium plate and r_p is the fraction of the incident power coupled into the meter. It must then be remembered that more than one reflected beam is responsible for the power observed at the meter since reflection can also take place at the back surface of the germanium plate as in Figure 3. An expression for r_p which includes both the multiple reflections and net absorption in the plate is

$$r_p(\theta) = \frac{r_s(\theta)\left[1 + (1-2r_s(\theta))\, e^{-2\alpha d(\theta)}\right]}{1-r_s^2(\theta)\, e^{-2\alpha d(\theta)}} \tag{6}$$

where the fractional power reflected off a single surface at an incidence angle ϕ and refracted angle θ is given by[9]

$$r_s(\theta) = \left[\frac{\tan(\theta-\phi)}{\tan(\theta+\phi)}\right]^2 \tag{7}$$

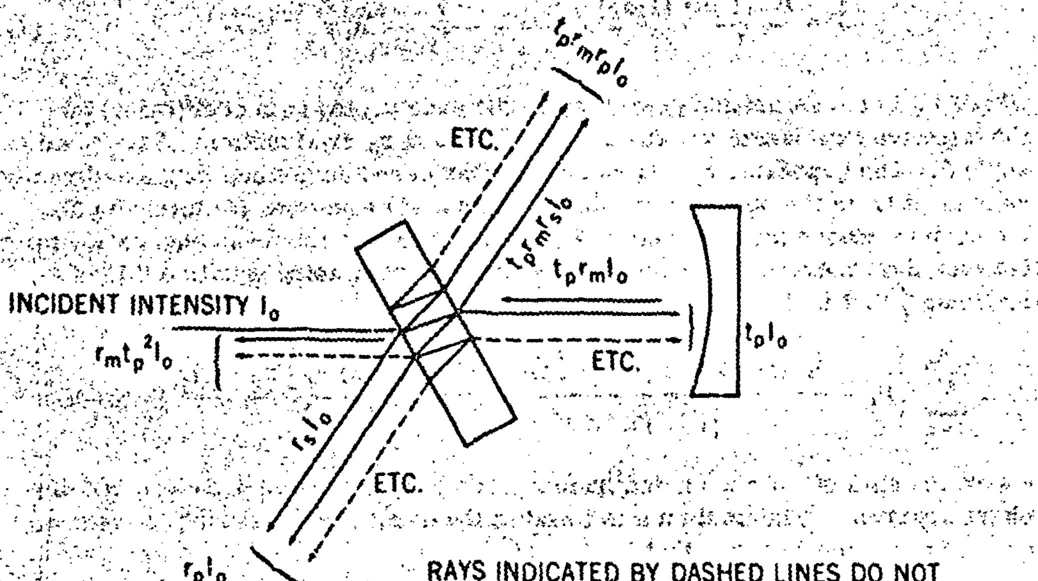

Figure 3. *Parameter definitions: r_p is the fraction of the incident intensity I_o coupled into the power meter whereas r_s is the partial contribution due to a reflection off a single surface of the coupling plate.*

and the absorbing length per pass in the plate is

$$d(\theta) = t/\cos\phi \tag{8}$$

where t is the plate thickness and the refractive angle ϕ is determined from Snell's Law. For germanium at 10.6 micrometers, the index of refraction is equal to 4.003[10] and the power coefficient is 0.055/cm[11]. The transmission of the germanium plate is given by

$$t_p = \frac{\left(1-r_s\ (\theta)\right)^2\ e^{-\alpha d(\theta)}}{1-r_p^2\ (\theta)\ e^{-2\alpha d(\theta)}} \tag{9}$$

and the total absorption in the plate can be found by substituting equations (6) and (9) into the expression

$$a_p = 1-r_p-t_p \tag{10}$$

It can be shown using Rigrod's results[7] and Figure 2 that the power entering the meter is given by

$$P = \frac{P_s\ (1-a_j)\ (1-a_b)\ \sqrt{r_2}\ \left[g_o\ell + \ell n\ \sqrt{r_1\ (\theta)\ r_2}\right]\ r_p\ (\theta)}{\left[\sqrt{r_1\ (\theta)} + \sqrt{r_2}\right]\ \left[1-\sqrt{r_1\ (\theta)\ r_2}\right]} \tag{11}$$

where P_s is the saturation power, g_o is the unsaturated gain coefficient, ℓ is the effective gain length and the quantities r_1 and r_p are functions of the plate angle θ. The quantities r_1 and r_p can be considered data since they are directly calculable from the observed incidence angle θ via equations (5) through (10). We wish to choose the parameters P_s and g_o such that the mean square deviation between the observed power and the power calculated using equation (11) is a minimum, that is

$$S \equiv \sum_{i=1}^{n} \left[P_i - \frac{P_s\ (1-a_j)\ (1-a_b)\ \sqrt{r_2}\ \left[g_o\ell + \ell n\ \sqrt{r_1\ (\theta_i)\ r_2}\right]\ r_p\ (\theta_i)}{\left[\sqrt{r_1\ (\theta_i)} + \sqrt{r_2}\right]\ \left[1-\sqrt{r_1\ (\theta_i)\ r_2}\right]}\right]^2 = \text{minimum}$$

where the sum is over the n data points taken at a particular pressure and discharge current. The minimum is found in the usual way by setting the partial derivatives equal to zero, that is

$$\frac{\partial S}{\partial P_s} = \frac{\partial S}{\partial g_o} = 0 \tag{12}$$

The above conditions lead to the following expressions for P_s and g_o in terms of the observed data points:

$$P_s = \frac{1}{\sqrt{r_2}\,(1-a_j)\,(1-a_b)}\;\frac{S_1}{S_2 - T\,S_3} \tag{13}$$

and

$$g_o = -\frac{1}{\ell}\left(\ell n\,\sqrt{r_2} + T\right) \tag{14}$$

where

$$T \equiv \frac{S_2\;S_4 - S_1\;S_5}{S_3\;S_4 - S_1\;S_2} \tag{15}$$

and

$$S_1 \equiv \sum_{i=1}^{n} \frac{P_i\,r_p\,(\theta_i)}{\left[\sqrt{r_1\,(\theta_i)} + \sqrt{r_2}\right]\left[1 - \sqrt{r_1\,(\theta_i)\,r_2}\right]}$$

$$S_2 \equiv \sum_{i=1}^{n} \frac{r_p^2\,(\theta_i)\;\ell n\;\sqrt{r_1\,(\theta_i)}}{\left[\sqrt{r_1\,(\theta_i)} + \sqrt{r_2}\right]^2\left[1 - \sqrt{r_1\,(\theta_i)\,r_2}\right]^2}$$

$$S_3 \equiv \sum_{i=1}^{n} \frac{r_p^2\,(\theta_i)}{\left[\sqrt{r_1\,(\theta_i)} + \sqrt{r_2}\right]^2\left[1 - \sqrt{r_1\,(\theta_i)\,r_2}\right]^2} \tag{16}$$

$$S_4 \equiv \sum_{i=1}^{n} \frac{P_i\,r_p\,(\theta_i)\;\ell n\;\sqrt{r_1\,(\theta_i)}}{\left[\sqrt{r_1\,(\theta_i)} + \sqrt{r_2}\right]\left[1 - \sqrt{r_1\,(\theta_i)\,r_2}\right]}$$

$$S_5 \equiv \sum_{i=1}^{n} \frac{r_p^2\,(\theta_i)\;\ell n^2\;\sqrt{r_1\,(\theta_i)}}{\left[\sqrt{r_1\,(\theta_i)} + \sqrt{r_2}\right]^2\left[1 - \sqrt{r_1\,(\theta_i)\,r_2}\right]^2}$$

A computer program calculated the best estimate of P_s and g_o from the data based on the algorithm provided by equations (13) through (16) and then plotted the theoretical curve (using the best estimates) against the observed data. One difficulty in applying this technique to the waveguide laser was the uncertainty in the amount of coupling loss between the resonator and the guide presented by the terms a_j and a_r in equation (4) and the resulting uncertainty in r_2 which appears as a "known" quantity throughout equations (16). This difficulty would

be virtually non-existent in a similar study of gain and gain saturation characteristics in standard lasers where the other losses appearing in equation (4) can be estimated reasonably well. To circumvent this problem, r_2 was allowed to vary through a wide range (0.85 to 0.98). The average mean square deviations obtained with each value of r_2 for each of the data sets corresponding to a particular pressure and current setting were then compared. This procedure indicated that $r_2 = 0.96$ was a reasonable choice that allowed good agreement between theory and experiment over the total data set. Since the reflectivity of a gold mirror is about 98-percent at 10.6 microns[12], this leaves an additional loss of 2-percent due either to imperfect coupling between the waveguide and resonator or to the astigmatic effect of the Brewster window since the other losses, such as diffraction losses and absorption in the Brewster window, are much smaller in magnitude. The sample graphs in Figure 4, showing the agreement between experiment and theory, were obtained at an average tube pressure of 35.2 torr. The different curves correspond to different discharge currents.

IV. RESULTS AND DISCUSSION

Figure 5 shows the dependence of the unsaturated gain coefficient on current and pressure. At an average tube pressure of 20.2 torr, the small signal gain appears to have a broad peak in the current range between 3 and 5 milliamps. The curves at 28.0 and 35.2 torr almost coincide, indicating a probable maximum of the small signal gain in this pressure region. This is further suggested by the fact that as the pressure was increased to the 70 torr range, the power level was seen to decline. Bridges[4] et al. observed a peak small signal gain at a pressure of 40 torr for a 1-mm diameter bore waveguide CO_2 laser at 20° C. Perhaps more significant is the fact that, at the higher pressures, the small signal gain decreases monotonically as the discharge current is varied upwards from 3 milliamps. Clear experimental evidence for this is the steady narrowing of the power curves in Figure 4 as one goes to higher currents. Thus, the unsaturated gain coefficient, which initially increases with increasing current[2, 15-17] peaks at current levels below 3 milliamps in the waveguide laser as compared to a typical value of 10 ma[2, 14, 15] in standard CO_2 laser amplifiers. The decrease in gain at higher currents has been attributed to an increase in the axial gas temperature.[13-14] It is clear then, from the experimental data presented, that in the case of glass or pyrex capillary tubes (which have a high radial thermal impedance), gas temperature is a serious limiting factor. Bridges[4] et al. have suggested the use of high thermal conductivity BeO ceramic capillaries. A greater portion of helium in the gas mixture might also alleviate the problem to some extent.

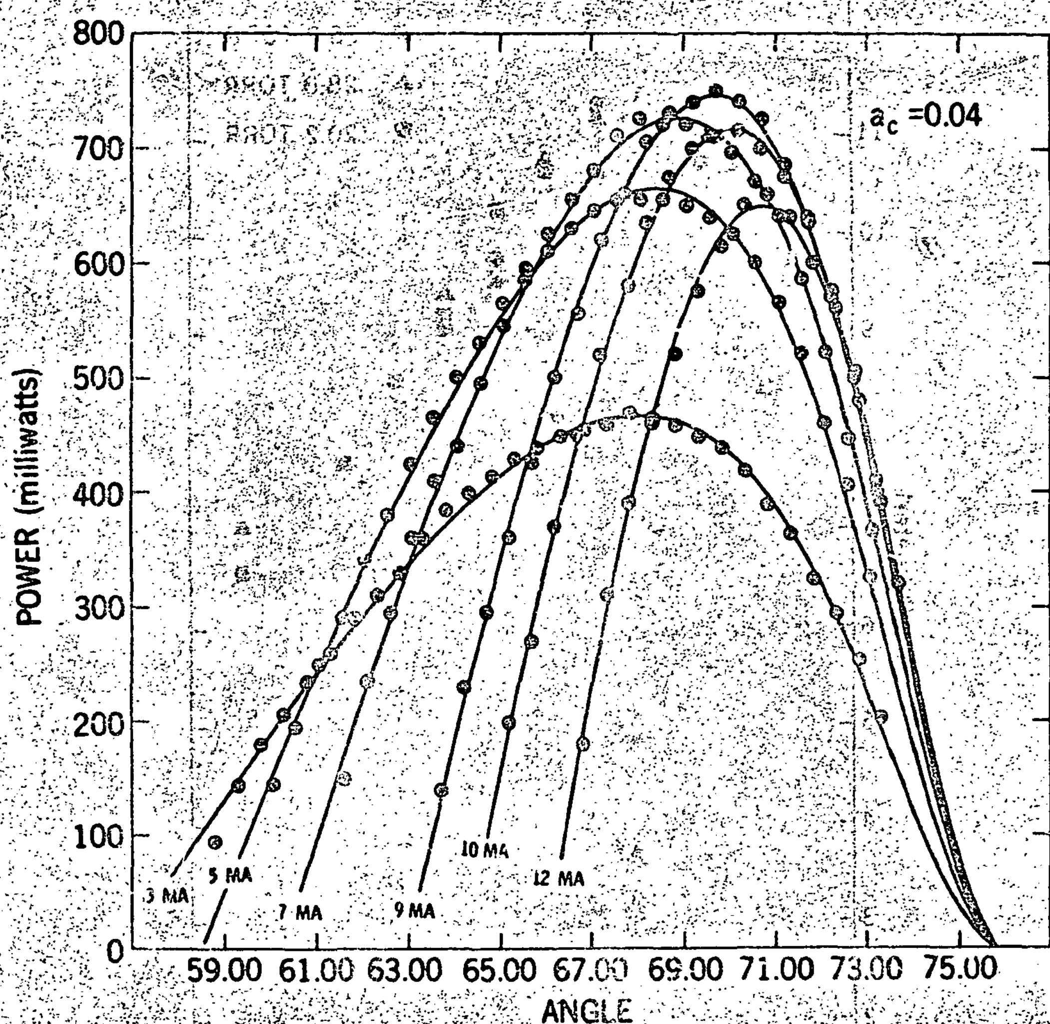

Figure 4. Power coupled into the meter versus plate angle of incidence for an average tube pressure of 35.2 torr and several values of discharge current. The solid lines correspond to the theoretical curves generated using the estimates of g_0 and P_s obtained from the least-squares algorithm derived in the text. A one-way dissipative loss of 4% (exclusive of coupling plate loss) has been assumed. The total power coupled out of the cavity is approximately twice that shown in the figure for a given plate angle. The Brewster angle for germanium is 76°.

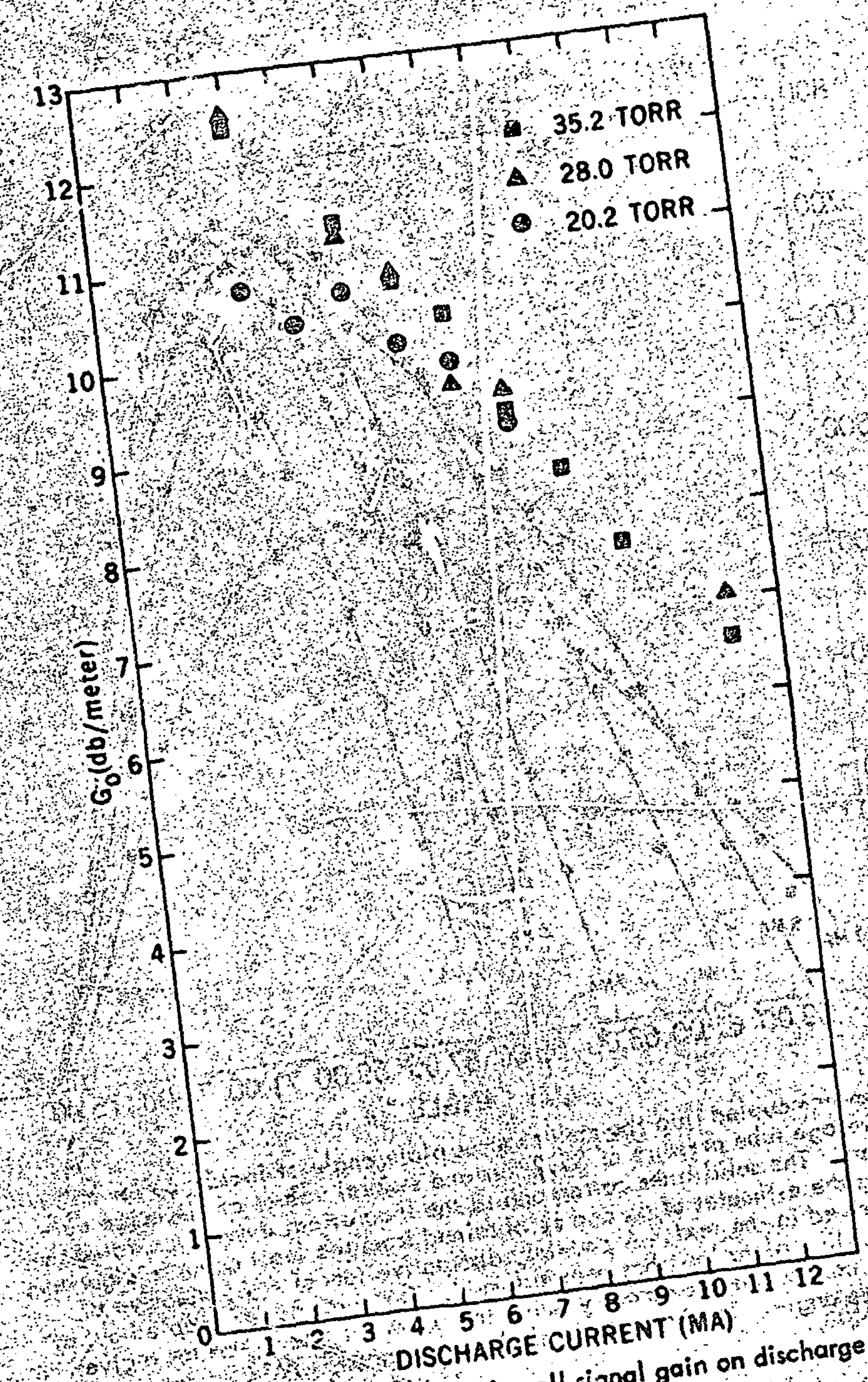

Figure 5. Dependence of small signal gain on discharge current and pressure.

Figure 6 illustrates the dependence of the saturation intensity on discharge current and tube pressure. To obtain the values shown, the saturation power P_s derived from the data has been multiplied by a factor of 118 to take into account the zero-order Bessel Function field distribution of the dominant HE_{11}[18,19] waveguide mode. Although the saturation parameters reported here would be considered large by the usual standards[20-23], they are consistent with the rough value of 2100 W/cm^2 previously reported by Bridges[4] et al., for a 1-mm diameter bore waveguide CO_2 laser at 20°C. The latter authors suggest that the large values of saturation parameter may be due primarily to reduction of lifetimes by wall collisions and to the short dwell time of the gas in the active region, rather than to a simple increase in pressure and the resultant increase in the collisional relaxation rates. The data in Figure 6 indicates, however, that at a typically optimum operating current of 8 ma for this pressure range and tube, the saturation parameter is a strong function of pressure and rises at a rate of 50 watts per cm^2 per torr if one assumes a linear relationship between saturation parameter and tube pressure at constant discharge current. This would imply a saturation parameter of about 500 watts/cm^2 at a pressure of 10 torr where a large number of measurements have been made.[24] The latter measurements were taken, however, at higher discharge currents and in large bore amplifier tubes. Christensen[24] et al., and also Smith and McCoy[25] have cautioned that, due to diffusion of excited CO_2 molecules into and out of the beam, the measured saturation intensity is inversely dependent on the probe beam radius. In the present experiment, the probe beam radius is determined by the 1/e power points of the dominant HE_{11} waveguide mode which, for a 2-mm diameter capillary, yields a rather small effective radius of 0.55-mm. Extrapolation of the results in reference 24 indicate that the use of a 0.55-mm probe beam would have yielded a value as high as 200 watts/cm^2 at the 10 torr pressure - a value still substantially smaller than the projected value of 500 watts/cm^2 based on extrapolation of the present data. The discrepancy is further compounded by the fact that the data in reference 24 was taken at a relatively high discharge current of 26 ma. It would seem, therefore, that radial diffusion of excited CO_2 molecules into the beam, although important, cannot totally account for the large increase in saturation parameter. This is further evidence that the large saturation intensities observed for waveguide CO_2 lasers are heavily dependent on wall collisions and/or rapid flow rate as suggested by Bridges and his co-workers.[4]

The optimum discharge current was observed to rise from 6 ma at 20.2 torr to 9 ma at 35.2 torr and then decrease again to a value of about 3 ma as the pressure was increased to roughly 90 torr. The power level (from both sides of the coupling plate) dropped to 150 milliwatts at the higher pressures from its peak of 1.5 watts at a pressure of 35.2 torr. At the peak power point, the tube voltage was 4900 volts and the discharge current was 9 ma.

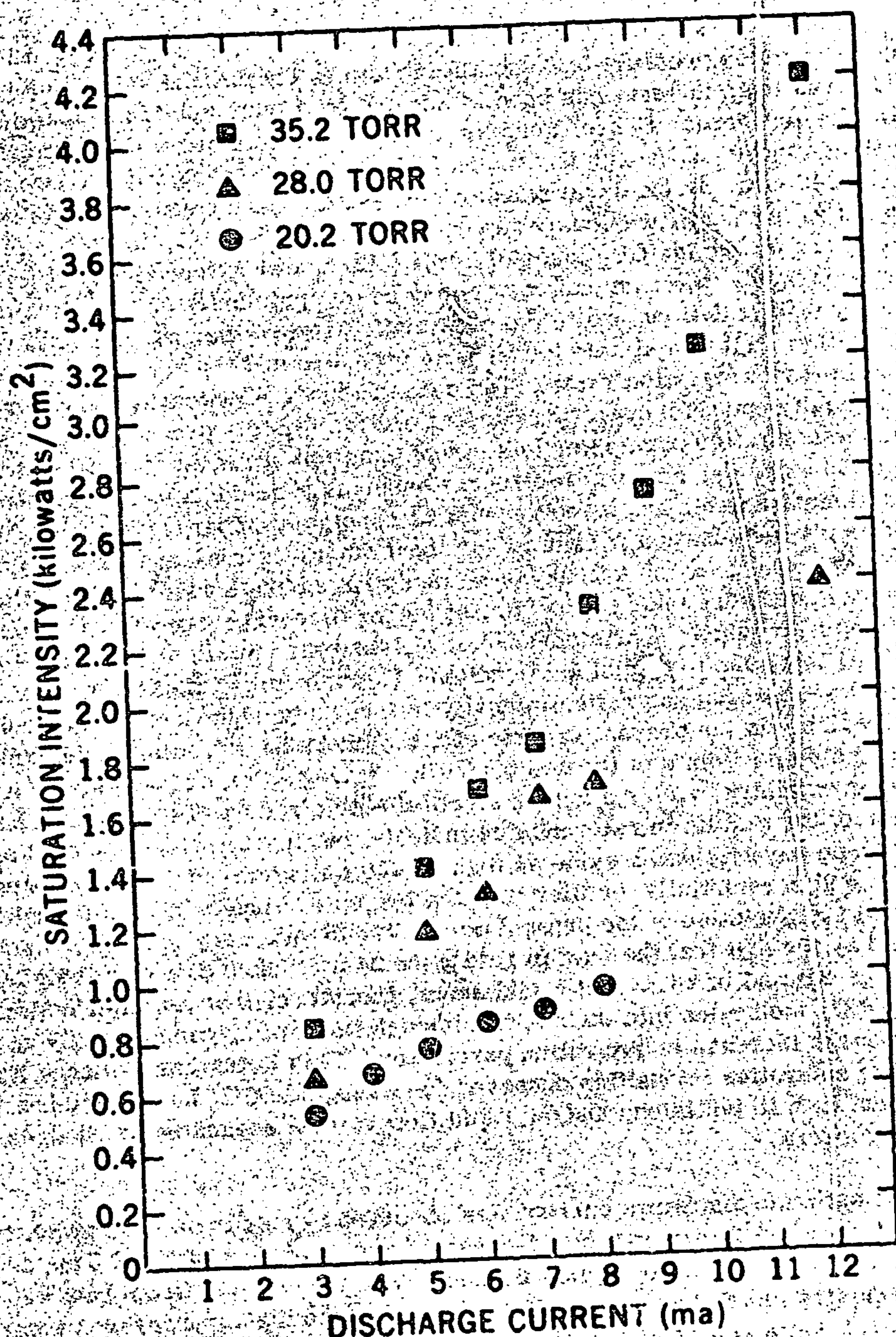

Figure 6. Dependence of saturation intensity on discharge current and pressure.

The numerical values for the unsaturated gain coefficient and saturation parameters reported here were used to predict the power output (at optimum coupling) expected for a shorter tube (shown in Figure 7) at a particular current of 6 ma and pressure of 35.2 torr. The second tube, which also had a 2 mm bore, was assumed to have a gain length of 5 cm (includes capillary and 5.7° flares) and a one way dissipative loss of 4% as estimated previously. The predicted maximum was 307 milliwatts while the observed maximum was 302 milliwatts. The agreement is remarkably good when one considers the rather arbitrary definition of gain length and the fact that the flow rates varied slightly in the two tubes. The small tube was operated at average pressures as high as 68 torr where the power level dropped to 145 milliwatts at an optimum current of 4 ma.

V. CONCLUSIONS

Measurements of the small signal gain and saturation intensity of a waveguide CO_2 laser under various pressure and discharge current conditions have been made. The method is an extension of the variable loss approach used by Witteman[12] and makes full use of the Rigrod equations for gain saturation by allowing the estimates of g_o and P_s to be based on an arbitrarily large number of data points. The gain measurements clearly indicate that, for pressures greater than 28 torr, temperature effects degrade the small signal gain at discharge currents as low as 3 milliamps. The unusually high saturation intensities cannot be explained solely by pressure and diffusion effects.

ACKNOWLEDGEMENTS

The authors are indebted to R. Jensen and M. Tobin for data which was of great assistance in the design and construction of the waveguide CO_2 laser described here. W. B. Bridges first described the potential of the waveguide CO_2 laser to the latter two authors in July 1971. The work described in this paper was begun in January 1972 when it became evident that a number of laboratories had initiated work in this area.

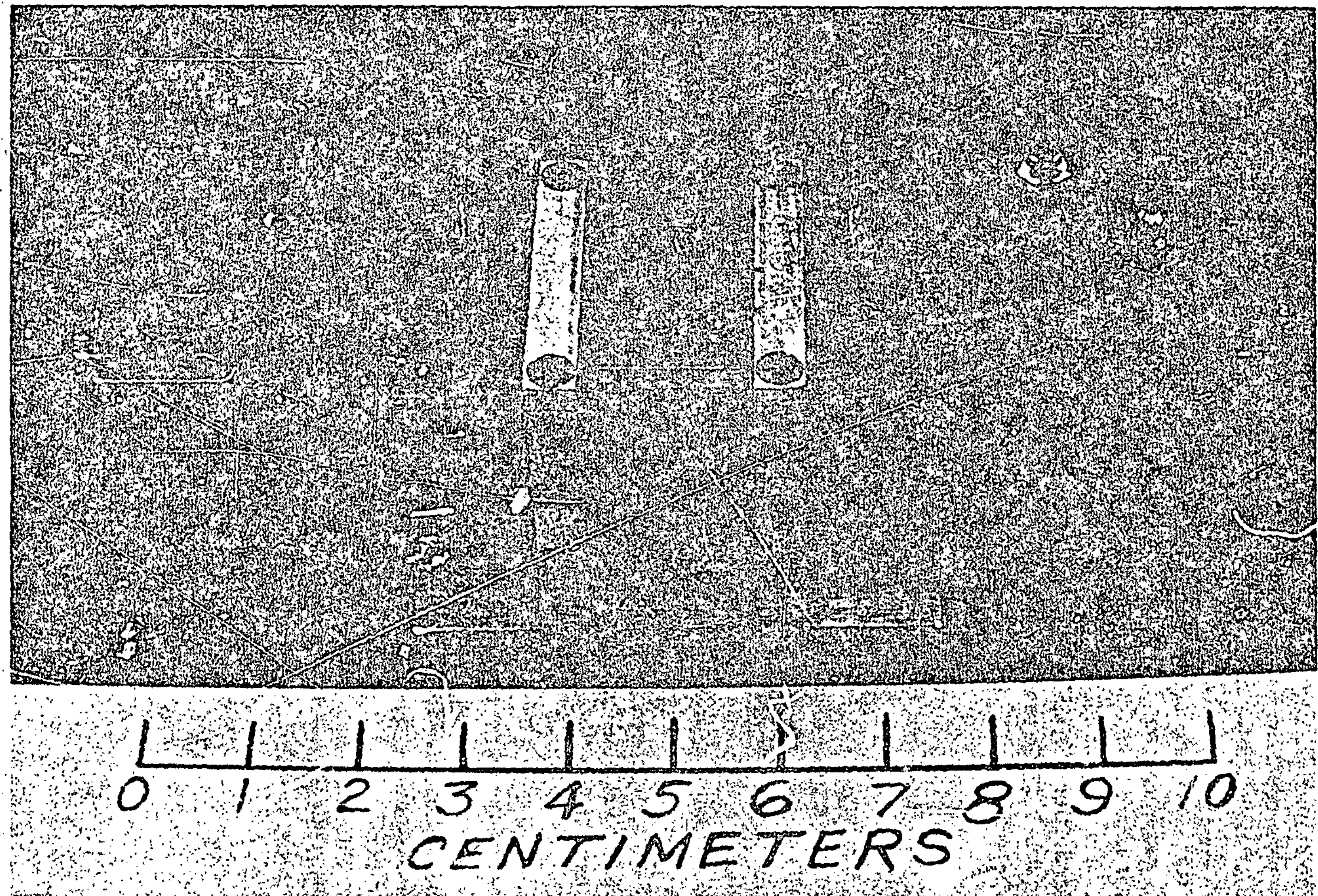

Figure 7. A waveguide laser tube having a 2-mm diameter bore, a capillary length of 3-mm, and 5.7° flare angles each one cm in length.

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